AFFINE STATISTICAL MANIFOLD: FINITE STATE SPACE

GIOVANNI PISTONE

ABSTRACT. An affine space, as defined by H. Weyl, is a triple $(M, V, \to)$, where V is a real vector space and M is a set with a binary external operation $M^2 \ni (P,Q) \mapsto \overrightarrow{PQ} \in V$ such that the parallelogram law holds. This notion was introduced in Relativity in order to derive simple notions of velocity and acceleration for curves in non-flat spaces. I have extended this setup to vector bundles to discuss the relevant affine geometries of the probability simplex. This type of study was called Information Geometry by S-i Amari. Especially, Amari and Nagaoka introduced a crucial notion of duality between mixture families and exponential families. J. Aitchinson developed a similar geometrical treatment of what he calls Statistics of Compositional Data. This affine setup is compatible but different with the metric set due to CR Rao and based on the observation that the Fisher information matrix of a statistical model defines a Riemannian metric on the probability simplex. A previous lecture on similar topics is transcribed in [8]. The present lecture is based on that and material taken from [10, 7, 4]. The continuous case, not treated here, requires particular technical notions of differential geometry and functional analysis. See a summary in [9].

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DE CASTRO STATISTICS, COLLEGIO CARLO ALBERTO
URL: `https://www.giannidiorestino.it`