Affine Geometry of the Statistical Bundle

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The Statistical Bundle is the set $S\mathcal{E}$ of couples (p,u) with p strictly positive probability function and u a real random variable such that $E_p(u) = 0$. It is a vector bundle $\pi\colon S\mathcal{E} \to \mathcal{E}$ where $\mathcal{E}$ is the open probability simplex on a finite set X. For example, if $\theta \mapsto p(\theta) \in \mathcal{E}$ is a smooth one-dimensional probability model, the lift $\theta \mapsto (p(\theta), Dp(\theta))$ is a smooth curve in the Statistical Bundle, where $Dp(\theta)$ is the Fisher's score (logarithmic derivative) of the model.

Given two points (p,u) and (q,v) in $S\mathcal{E}$, one can define affine displacements in the elementary sense of Weyl (1921),

$$((p,u),(q,v)) \mapsto V_{p,u}(q,v) \in S_p\mathcal{E},$$

and correspondingly define an affine geometry on the Statistical Bundle. The further assignment of a duality pairing on the fibres produces by dualization a dually flat geometrical structure. See a tutorial in G Chirco and G Pistone arXiv:2204.00917.

Defining the affine geometry on the Statistical Bundle implicitely defines the connection on the non-parametric affine bundle of the open probability simplex.

The study of Information Geometry of the Statistical Bundle has other distinct advantages—first, a simplified presentation of the transport Problem of the probability simplex. See G. Pistone. Statistical bundle of the transport model. In GSI 5th Proceedings, 752–759. Springer-Verlag, 2021. Second, the vector bundle and its dual provides the proper setting for studying Lagrangian and Hamiltonian mechanics of the probability simplex. See G Chirco, L Malagò, G Pistone. Lagrangian and Hamiltonian dynamics for probabilities on the statistical bundle. *International Journal of Geometric Methods in Modern Physics*, 19(13):2250214.1–46, August 2022.

The talk will mention other relevant references, particularly the generalization to continuous state space. My presentation will mainly focus on the statistical meaning of geometric concepts.