Information geometry of the Gaussian Space

G. Pistone

de Castro Statistics, Collegio Carlo Alberto, Piazza Arbarello 8, 10122 Torino

Abstract

This talk is based on the conference papers [1,2,3]. It presents an overview of the topic and some of the current developments.

The exponential manifold [4,5] on the finite-dimensional Gaussian space [1] has special features namely, the existence of a finite entropy and finite moments of all orders for all densities in the manifold. Moreover, it is possible to discuss the continuity of translations, Poincareé inequalities, and the generalized differentiability for densities. As a consequence, it is possible to define an exponential manifld for densities belonging to a given Orlich-Sobolev space with Gaussian weight.

A field of application is the study of the dimensionality reduction for of evolution equations in the sense of D. Brigo [2] i.e., the projection of the solutions onto a finite-dimensional exponential family.

The basic exponential representation of densities in the exponential manifold can be modified by the use of the so-called deformed exponentials for example, the Nigel Newton exponential [6]. The linear growth of the deformed exponential allows for a simplified treatment of the manifold of densities in a Sobolev space with Gaussian weight.

References

1. Pistone G. (2018) Information Geometry of the Gaussian Space. In Ay N., Gibilisco P., Mats, F. Eds *Information Geometry and Its Applications.* Springer 119–155.

2. Brigo D. and Pistone G. (2017) Dimensionality reduction for measure valued evolution equations in statistical manifolds. In Nielsen F., Critchley F., Dodson C.T.J. Eds *Computational Information Geometry.* Springer 217–265.

3. Montrucchio L. and Pistone G. (2017) Deformed exponential bundle: the linear growth case. In Nielsen F., Barbaresco F. Eds *Geometric Science of Information.* Springer pp. 239-246.

4. Pistone G. and Sempi C. (1995) An infinite-dimensional geometric structure on the space of all the probability measures equivalent to a given one. *The Annals of Statistics* **23** 1543–1561.

5. Santacroce M., Siri P. and Trivellato B. (2016) New results on mixture and exponential models by Orlicz spaces. *BERNOULLI* **22**(3) 1431–1447.

6. Newton N. (2019) A Class of Non-Parametric Statistical Manifolds modelled on Sobolev Space. arXiv:1808.06451v4.